

Split or steal or gift: Nash equilibria under altruistically extended payoffs

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Incentives for cooperation can come from different conceptual directions, beyond the “sticks and carrots” duality. This paper looks into potential, altruistic extensions of the payoff structure known from the prisoner's dilemma in game theory. The illustration of the payoff extensions happens through the derived game “split or steal”. The notion of altruism has been frequently and increasingly channeled into game theory previously, yet no attempts were found where it affects the design of the payoff matrix directly and explicitly. The aim of this paper is to show that the original payoffs of the game “split or steal”, and of the prisoner's dilemma more broadly, do not cover the full spectrum of human behavior in the strategic dimension of the situation, and that an altruistic adjustment, the “giving as a gift” option, gives space to corresponding incentives. The results are interpreted to be applicable to a number of real-life situations, complex, as well as competitive, both in the corporate and the governmental sphere.

Keywords: game theory, Nash equilibrium, altruism, gift giving, reward scheme

1. Introduction

The game “split or steal” is based on a prisoner’s dilemma type payoff structure. In its conventional form, it gives rise to a situation with a 2-by-2 payoff table. Potential expansions have been studied, but the columns and rows therein focus on the inclusion of additional actors (up to indefinite, n), as well as on neutral reactions.

Through this paper, I aim to illustrate that the traditional “split or steal” game does not cover the entire spectrum of individuals’ propensity to share, and rewards players in a partially distorted way, as compared to situations in real life. I focus on altruistically extended payoff schemes, in which I expand the number of options available to 3, with the possibility of giving as a gift, thus increasing the payoff table to nine (3x3) substantial cells.

The purpose is to support the stance that with a modified reward system, people’s tendency to share and to give gifts would increase. In practical terms, this may provide ground for adjusting economic incentives and reward schemes. My research serves to prove the *raison d’être* of frameworks that give space to altruism, through simulational methods. Results may lead to rethinking the framework of rewards used in complex, and even competitive situations – both in the business and the public sphere. My approach and aims combined are unique, as reflected by searches conducted¹.

¹ On December 10, 2023. Google Scholar yields absolutely no results for “3 by 3 payoffs” (searched with quotation marks inclusive). As for “3 by 3 payoff”, there are only 14 results, with only 9 of these mentioning Nash equilibria, or the name Nash in general. Each of the 9 remaining articles is either applying 3 by 3 payoffs in a highly specialized subcontext, is descriptive of existing phenomena in such contexts, or is general, but does not mention altruism.

Three central questions have been formulated: one concerning the payoff designs of “split or steal or gift” games (architecting the tables); the second about participant behaviors in games of such frameworks; and the third regarding the overall connection of extended payoffs with practice.

RQ1: What are the rules, ratios, or values (to be written in the additional five fields of the payoff table) in altruistically extended “split or steal or gift” games that are likely to increase participants’ altruistic choices (shifting Nash equilibria toward the cell representing mutual gifting) whilst retaining the game nature of choices?

H1: I predict that the combinations in the additional payoff cells...

- H1a: ... cannot be organized along a fixed sum,
- H1b: ... have to reflect additional layers of tension and risk,
- H1c: ... have to reflect that altruistic behaviors need to be rewarded, be worthwhile overall,
- H1d: ... have to be calibrated so as to result in maximum payoff values in the case of both players choosing the “gift” option.

RQ2: Do individual, simulated players in the adjusted, altruistically extended payoff schemes of the “split or steal or gift” game behave more selflessly than in the standard “split or steal” version of the game?

H2a (null-hypothesis): No, they do not.

H2b: Yes, they do.

Both RQ1 and RQ2 are methodological in nature, thus their validation is primarily addressed within the methodology section. RQ2 is the part that the simulations will focus on.

RQ3: In what ways is an altruistically extended payoff scheme more realistic than the split or steal game?

H3: In practice, selflessness is rewarded beyond fixed sum structures, which can be witnessed both in the corporate and the governmental sphere.

My paper is structured as follows. Section 2 provides the theoretical background. Section 3 describes the concrete framework, the tools, and the conduct of the methodology. Section 4 summarizes results, which are then broadly discussed in Section 5. Section 6 describes further research opportunities, and section 7 concludes the paper.

2. Theoretical Background

In the formulation of Daskalakis et al., “Game Theory is about the strategic behavior of rational agents.”, and games are “thought experiments modeling various situations of conflict” (2009, p. 89). More broadly: “In game theory, a game may be any situation in which there are interdependent decisions, and the players are all the decision-making

entities” (Elridge 2023). Aligning with these definitions, the subsections look into the strategic aspects of equilibria, into how and in what context the “split or steal” game has been examined thus far, as well as into altruistic extensions of games, a procedure not independent of the altruistic extensions of human behavioral models. For the compilation of works for the review, a mixed technique of Google Scholar searches and collection through reference lists of the works already listed has been used.

2.1. The Nash Equilibrium

The notion of the game forks off into two distinct categories: cooperative and non-cooperative games. Cooperative games are characterized by possibilities: “In a cooperative game, players may work together by forming groups, so-called coalitions, and may take joint actions so as to realize their goals better than if they were on their own” (Rothe 2021, p. 15073). For non-cooperative games, the emphasis is on the constraints: “A game is non-cooperative as long as no mechanism exists for the players to make binding agreements with one another” (Elridge 2023).

For non-cooperative games, there exists a situation in which the individual players cannot improve their expected outcome by changing their own strategy (Elridge 2023). This is called the *Nash equilibrium*, named after the late American mathematician John Forbes Nash Jr., who introduced its definition and characteristics in his seminal 1950 article titled “Equilibrium points in n -person games”. The Nash equilibrium can be a single cell in a given payoff structure, however, alternatively, “[i]t is possible for there to be multiple Nash equilibria to a given problem” (Elridge 2023).

From the many non-cooperative games known and described to date, the choice for applying and testing altruistic extensions in payoffs has fallen here upon the structure of the widely researched and discussed as the “prisoner’s dilemma”. The game theoretic model stems from 1950, developed in the frame of an experiment by the mathematicians Melvin Dresher and Merrill Flood, with the criminal narrative having been added by Albert Tucker, Nash’s thesis advisor (Holt–Roth 2004, p. 4000).

The prisoner’s dilemma² is one of the widest known instruments in game theory, with the first comprehensive empirical work dating back to 1965 (Rapoport et al. as referenced in Capraro–Perc 2021, p. 3). Within the prisoner’s dilemma model, and according to a self-interested and rational model of the individual, without the chance to communicate, both prisoners will prefer the option of confession to the option of silence, this being the Nash equilibrium.

² The background narrative is of two prisoners who have been arrested for the same crime. Individually, both care more about their personal freedom than about the welfare of their accomplice. The prosecutor offers each of them, separately, the choice to confess (i.e. defect, in the context of the game) or to remain silent (i.e. cooperate, in the context of the game). If one confesses and the other remains silent, the one who confessed goes free immediately, while the other serves a longer (e.g. 10 year) prison sentence. If both confess, both will have to serve a moderate (e.g. 5 year) prison sentence. In the case of both of them remaining silent, the prison sentences will both be relatively short (e.g. 2 years). “The ‘dilemma’ faced by the prisoners here is that, whatever the other does, each is better off confessing than remaining silent. But the outcome obtained when both confess is worse for each than the outcome they would have obtained had both remained silent” (Kuhn 2019).

The reason this underlying structure will be of use in this study is twofold. For one, “[...] the puzzle illustrates a conflict between individual and group rationality” (Kuhn 2019). The reason is that “[...] if the payoffs are not assumed to represent self-interest, a group whose members rationally pursue any goals may all meet less success than if they had not rationally pursued their goals individually” (Kuhn 2019). Second, the dilemma highlights “[...] a choice between selfish behavior and socially desirable altruism” (Kuhn 2019). The individual versus the group, as well as selfishness versus altruism are notions separated by economically permeable boundaries – different interpretations of individual behavior, with different contexts (in terms of emphases and extension) allow the model of the agent to pass from one side of these two dimensions to the other.

2.2. “Split or steal” games as experiments in economics

One of the many variations in applying or “translating” the prisoner’s dilemma is the game called “split or steal”. A reward is to be shared equally, taken entirely by one of the two players, or to be lost completely, depending on the participants’ choices. The mechanism and reward structure (in terms of ordinality) is analogous to that of the prisoner’s dilemma.

Sticking to the rational, self-interested model of agents, it is easy to see the following: “The dominant strategy in this game is to always pick steal, since this maximizes profit if the other picks split and doesn’t matter if they steal. The Nash Equilibrium in this game is for both players to walk home with nothing” (Cornell University 2014).

Beside analogous laboratory experiments, the first wider appearance that the game made can be dated to the early 2000’s, when the entertainment industry began utilizing its mechanism. Best known for this was the British television show *Golden Balls*, running from June 2007 to December 2009 (van den Assem et al. 2012, p. 4). Van den Assem et al. provide a succinct description of the part of the show that is relevant to this paper: “In the final stage of *Golden Balls*, contestants make a choice on whether or not to cooperate in a variant of the famous prisoner’s dilemma. In particular, the two final contestants independently have to decide whether they want to ‘split’ or ‘steal’ the jackpot. If both contestants choose split, they share the jackpot equally. If one chooses split and the other chooses steal, the one who steals takes the jackpot and the other gets nothing. If they both steal, both go home empty-handed” (van den Assem et al. 2012, p. 3).

The (online) video game named *Split or Steal* is an iterated version of the phenomenon, involving real players interacting with each other, and collecting “karma”, based on the trustworthiness of the behaviors they exhibit. The game design contains elements of inter-player and global communication too, resulting ultimately in “an absurd alchemy that combines social engineering, free-to-play incentives, and idle-game satisfaction that scratches some itch at the back of your brain” (Feldman 2020). As of 2020, split choices in this game were reported to be around 81% high (Feldman 2020), a figure substantially higher than detected through televised game shows (as will be discussed below).

One of the seminal works in approaching the “split or steal” game as an economic experiment is the above-mentioned 2012 article “Split or Steal? Cooperative behavior when the stakes are large”, authored by Martijn J. van den Assem, Dennie van Dolder, and Richard H. Thaler. They state that the factor determining their choice falling upon a

television show to study the phenomenon is primarily the size of potential rewards, which is remarkably bigger than the common values used in laboratory experiments, and is also varying within a higher range (van den Assem et al. 2012, p. 2). However, they note an important difference between the “original” prisoner’s dilemma and the final stage of *Golden Balls*: “Where in the classic form of the prisoner’s dilemma defecting strictly dominates cooperating, here defecting only weakly dominates cooperating: choosing steal always does at least as well, and sometimes better than choosing split” (van den Assem et al. 2012, p. 3). The average cooperation rate turns out to be 53% (van den Assem et al. 2012, p. 4), lower than in the above-mentioned online game. The main conclusion of the article is the influential nature of context on attitudes (van den Assem et al. 2012, p. 16).

By a slight contrast with the van den Assem article, a 2009 paper reported a rate of 48% for cooperation in the *Golden Balls* show (Coffey 2009, p. 2), albeit the data in this paper is restricted to coming from episodes broadcast in 2007 (Coffey 2009, p. 7). Cooperation has been measured along identical and different genders, as well as similar and more distant age groups. Coffey’s paper contrasts with van den Assem et al. (2012) also in reporting men’s cooperation rate to be higher than that of women (Coffey 2009, p. 2). The paper highlights important parallels with the classical prisoner’s dilemma, such as the simultaneous nature of decisions to be made, but emphasizes important differences too, such as the zero-sum nature of “split or steal”, which cannot be said about the standard prisoner’s dilemma (Coffey 2009, p. 6).

Behavioral aspects have been taken into account heavily by Darai and Grätz, in their working paper “Golden balls: A prisoner's dilemma experiment” (2010). In the late 2010’s the split or steal element of *Golden Balls* has attracted even further scientific inquiry, going into specific details regarding pre-decision communication between contestants (Turmunkh et al. 2017, p. 1), and regarding a peculiar strategy of lying in one distinct episode, that had counterintuitive results of cooperation (Brams–Mor 2019).

2.3. Altruistic extensions in game theory

When speaking of altruistic extensions in the context of game theory, the process or phenomenon can be detected at two different but interconnected points in the literature. For one, works that examine individuals displaying altruistic behavior in standard game structures, and which thereby aim to broaden the economic understanding of humans. Second, the design of the classical game structures can be adjusted, so as to allow for altruistic behaviors to show explicitly. In the current subsection, I survey academic works in these two groups. This paper unifies the two strands in subsequent sections, in that both the preliminary assumption about individuals, as well as the game design have the notion of altruism included.

In 1993, the American behavioral economist Matthew Rabin published the article “Incorporating fairness into game theory and economics”, wherein he explores intersections of fairness equilibria (outcomes reflecting reciprocal motivations) and Nash equilibria. Through stylized facts, Rabin develops a framework to incorporate retributive and altruistic emotions into economic models, which he illustrates through the games *battle of the sexes*, the *prisoner’s dilemma*, and the *chicken* game. The payoff structures of these games require no alteration for the purpose of the paper though.

The economist David K. Levine's 1997 article "Modeling altruism and spitefulness in experiments" was among the earlier pieces of its kind. Levine tested his theory of altruism through a series of games, ranging from an *ultimatum experiment* to a *public goods game*, including a *competitive auction* and a *centipede game*. In terms of initial assumptions, his article relates to and relies upon the work of Rabin. As in many of the referenced articles, the author begins by conceptualizing altruism within a broad perspective. Throughout the chapters describing the experiments, as well as through a series of propositions, Levine supports his model of altruism in quantitative terms.

In a *Nature* review article in 2003, the Swiss economists Ernst Fehr and Urs Fischbacher emphasized the curious observation that "[d]epending on the environment, a minority of altruists can force a majority of selfish individuals to cooperate or, conversely, a few egoists can induce a large number of altruists to defect" (p. 785). They review results from classical games such as the ultimatum game, the dictator game, the prisoner's dilemma, and the public goods game, pushing their review to the limitations of altruism even, in the understanding of the given time period. Game theoretical models have not been altered at their core in their article either, but the interpretations provide the behavioral (and not the psychological) study of altruism with more space.

In his 2010 article "Mixed feelings: Theories of and evidence on giving", James Konow uses the dictator game framework in an experimental study to examine internal motivations for, as well as institutional effects on giving. He reports the results through a mixed interpretation of the categories of unconditional and conditional altruism. The dictator game is used in different variations, however, the differences in the game design serve the assessment of behavior in different contexts and relations, and represent no substantial changes or extensions to the structure of the game.

Roughly from the mid-2010s, one may observe the element of rationality being applied in studies that concern themselves with altruism's game theoretic analysis and interpretation. In their 2016 dictionary entry, Andreoni et al. highlight the rationality element, as well as the difficulty of capturing altruism, with the concept of *warm-glow giving* being a confounder in the process. The authors review a series of classical laboratory experiments of game theory: the prisoner's dilemma, the public goods game, dictator games, and trust games. In each of these, they "adopt the convention of using Nash equilibrium to refer to the prediction that holds if all subjects are rational money-maximizers" (Andreoni et al. 2016). What they reveal (through reviews) is that altruism is not a necessary condition of cooperation in the prisoner's dilemma, that in the public goods game, "with a dominant strategy of giving zero, any error or variance in the data could mistakenly be viewed as altruism" (Andreoni et al. 2016), that deliberate giving is not identical to altruism, that a "lack of social distance" between the researcher and the "dictator" may explain seemingly non-selfish behavior in the dictator game, and that in trust games with positive outcomes, motivation is reciprocal, and not altruistic (Andreoni et al. 2016).

The economists and regular co-authors Ingela Alger and Jörgen W. Weibull ask in their 2017 article "Strategic behavior of moralists and altruists" "whether altruism and morality help improve the material welfare properties of equilibria in strategic interactions" (p. 18). They get different responses depending on the conditions: the type of the game (public goods games vs. two-by-two games are tested), the length (static or repeated), and whether the focus is on altruistic, moral, or self-interested strategies (classes

of preferences) – which the authors clearly distinguish from the start. The authors point to the highly ambiguous relationship between the classes of preferences and welfare outcomes.

The article “Rational altruism” (Tóbiás 2023) is a peculiar example of treating altruism as endogenous to choices. The author explores the possibilities of pre-agreed degrees of altruism in the prisoner’s dilemma, and shows that the strictly dominant strategy (both players defecting) shifts as a result of allowing players to internalize the outcome of their opponents, in other words: to care about the other (Tóbiás 2023, p. 51).

As for methods of incorporating altruism in the game design directly, the German computer scientist Jörg Rothe has provided a prompt summary in 2021, mainly with the purpose of bettering real-world applications of AI simulations in the context of altruistic behavior (p. 15070). Rothe observes that “[f]rom the early beginnings of (non-cooperative) game theory due to von Neumann and Morgenstern (1944), a player (or agent) in a game has been viewed as a homo economicus [...]” (Rothe 2021, p. 15070) This perspective, however, as we saw in the works cited above, has been shifting. Not only has altruistic behavior been detected and used as an explanation in – mainly non-cooperative (Rothe 2021, p. 15071) – games, researchers have also aimed at changing games so as to introduce altruism into models of game theory directly.

Examples include interpreting altruism in utility functions, studying the efficiency of altruistic behavior (Rothe 2021, p. 15071; similar to the approach of Alger and Weibull 2017, above); assuming existing levels of altruism for players and looking for Nash equilibria under such conditions (in line with Tóbiás 2023); calculating minimum and optimum numbers of predefined altruists for certain desired outcomes (Rothe 2021, p. 15072); observing altruistic extensions of players’ preferences (Rothe 2021, p. 15073); and studying stability under altruistic extensions (Rothe 2021, p. 15074). None of the examples listed cover the option of altruism in the explicit and augmentative manner in which it is used in the present paper.

As a cautionary, clarifying, as well as contextual note, the following statements ought to be added to the review of altruistic extensions within game theory: “In light of rewards, the notion of altruism remains a controversial one. It is not to be interpreted as an act between individuals, for that would shift the process toward reciprocal exchange. It is an opportunity and procedure intrinsic to the system in its entirety, moving it – in the ideal case – towards a state of equilibrium.” (B. Hámori, personal communication, December 19, 2023, own translation from Hungarian).

3. Methodology

When designing payoff tables for the specific purpose of incentivizing altruism within games, a number of dimensions and conditions have to be taken into account. The baseline condition is to have a workable definition of altruism itself.

The altruistic extensions in game theory have no uniform underlying description of the concept in common, despite the relative nature of altruism – as compared to self-interested behavior – being evident. For the current game theoretic context, I am relying on the definition provided by Capraro: “A player may prefer to renounce to part of her gain in order to favor another player” (2013, p. 8).

Three-by-three (3x3) payoff tables have been used before in game theory, albeit for different purposes from mine. One may find them as tools of illustration for the so-called saddlepoint: “the outcome that rational players would choose in a two-person constant-sum game” (Brams–Davis 2023). The commonly known rock-paper-scissors game is a special case, without a saddlepoint (Duersch et al. 2010), yet a classic example itself for the 3x3 payoff structure for two players. Prisoner’s dilemma payoff structures have also been extended to 3x3 payoff tables to include the option, the third move of “neither” (beside “cooperate and “defect”), or of an “opt-out”, in the same manner – this latter version being labelled “optional PD” (Kuhn 2019).

In the paragraphs below, I have documented the thought process of designing *split or steal or gift* payoff tables, i.e. the way I arrived at the individual values in the cells, in the set of the sample tables.

For the sake of simplicity and clarity, the draft payoff designs will be for symmetric games only. Both participants face the same conditions, the same payoff structure, and have to make their decisions at the very same time, without prior communication.

The zero-sum (or fixed sum) vs. variable sum question is interrelated with the tensions of the game, for it is in variable sum games that the players have both common and opposed interests (Brams–Davis 2023). Certain areas of the payoff tables (such as the first three cells of the classical “split or steal” structure) may be described as internal zero-sum sections, but with the introduction of the additional “gift” layer, the ratio of the internal zero-sum area to the whole is about to decrease.

Whilst the tension, the dilemma nature of the game, is to be retained, there may arise a tension between this retention and designing payoffs so as to “steer” players into socially desirable outcomes. According to Kuhn (2019), “universal cooperation is the most socially desirable outcome”. The table has to be “tweaked”, so as to contain possibilities and to generate inclinations towards mutual altruism, even if that scenario cannot coincide with the dominant Nash equilibrium. To a certain extent, the attractive option to the individual player, leading to a Nash equilibrium – which tends to be located in the lower right corner, the absolutely non-cooperative section of the payoff table – may be counterbalanced by Pareto efficiency³ in the cooperative, or even fully altruistic (both choose to “gift”) equilibrium.

As for the power of the aforementioned tweaks: an incremental change in the payoffs may make the dominant strategy “trickle” towards the more cooperative outcomes, whilst keeping the dilemma nature and structure of the game. If we understand the trickling dynamically, it can stand for a potential *virtuous circle* of *backward reasoning*, more specifically, of *backward induction*.

I expect this positive mechanism to follow mainly from the structure of the payoffs itself, not only from the – realistically speaking – inherent altruism of the individual players. “In standard treatments, game theory assumes rationality and common

³ A quick definition: “A state of affairs is Pareto-optimal (or Pareto-efficient) if and only if there is no alternative state that would make some people better off without making anyone worse off” (Ingham 2023).

knowledge” (Kuhn 2019). The simulations will be standard treatments also in that players take the other players’ (likely and rational) strategies into account.

Without fixed-sum adherence, and with the intention to make choices trickle “northwest”, i.e. becoming more cooperative, then altruistic, the payoffs have to be designed with rewards overall increasing (or potential losses overall decreasing) towards the upper left, the altruistic corner of the payoff table.

The altruistic choice (in light of the potential strategies of the partner) must appear either risky but potentially more rewarding, or more safe but probably resulting in lower level rewards for any concrete choice, than in the cells “southeast”. This retains the tension. For the sake of clarity and simplicity, however, the differences between the payoff values here should be as low as possible, reflecting ordinality only.

According to the statements above, four payoff tables (see Tables 1 to 4 below) designed are about to follow. The first value in the cells is always the reward for player A (rows), the second for player B (columns). The core part, i.e. “split or steal” rewards have not been changed – the values in those cells are highlighted.

Table 1. The first payoff version for “split or steal or gift”

	B gifts	B splits	B steals
A gifts	2; 2	1; 2	0; 2
A splits	2; 1	1; 1	0; 2
A steals	2; 0	2; 0	0; 0

Source: own construction

Table 2. The second payoff version for “split or steal or gift”

	B gifts	B splits	B steals
A gifts	1; 1	0; 2	0; 2
A splits	2; 0	1; 1	0; 2
A steals	2; 0	2; 0	0; 0

Source: own construction

Table 3. The third payoff version for “split or steal or gift”

	B gifts	B splits	B steals
A gifts	2; 2	1; 3	0; 4
A splits	3; 1	1; 1	0; 2
A steals	4; 0	2; 0	0; 0

Source: own construction

Table 4. The fourth payoff version for “split or steal or gift”

	B gifts	B splits	B steals
A gifts	0; 0	2; 1	2; 0
A splits	1; 2	1; 1	0; 2
A steals	0; 2	2; 0	0; 0

Source: own construction

In my first payoff proposal (Table 1) for the “split or steal or gift” game, I have followed the logic that altruism generally brings higher rewards “to the table”, here – literally. A coincidence of “gift” choices doubles the total amount of rewards. This is in line with an excerpt from the originally Swedish saying “shared joy is double joy”. If one of the players gifts but the other one “only” splits, that is still a better outcome overall than just splitting mutually, or than any of the combinations involving a “steal” choice. Tension is retained by leaving the steal option “attractive” as it was, through the maximum payoff value for the player who steals, in case the other one chooses to gift.

In my second payoff proposal (Table 2) I have architected the table so as for it to remain fixed-sum (except for the last cell), with the sum value being 2. The design also incorporates a higher level of sacrifice that the player who chooses to gift is willing to make, by making mutual gifting equally rewarding to mutual splitting, yet numerically completely unrewarding if the other participant chooses any other option but to gift.

The next version (Table 3) turns the above explanation on its head. Here I raised individual and overall rewards to the maximum level of 4. Both players are better off if the other one gifts, and they split or steal, but by “just” splitting they can leave (or reward) their partner with at least 2. If, however, both players opt for this compromise of splitting, they will be worse off than they would have been with a mutual “gift” choice. The table could be described as a direct or linear extension to the original game, for it applies the very same algorithm.

Altruism can be rewarded in a more ambiguous way, too, in the cases that the other player does not exhibit such behavior. This is depicted in Table 4, where the mutual “gift” choice is just as disadvantageous as if both players were to steal, yet if one player gifts and the other splits or steals, the former one will earn a reward of 2.

From my initial RQ1 related subhypotheses (H1a, H1b, H1c, and H1d) the first three apply, that is, the payoffs (taking the whole table into account) cannot be organized along a fixed sum (H1a), have to reflect additional layers of tension and risk (H1b), and have to reflect that altruistic behaviors have to be rewarded in most cases (H1c). The fourth subhypothesis (H1d) has to be rejected, for – as illustrated in Tables 2 to 4 – mutual altruism does not have to be the cell of maximal payoffs.

It should be added that in the design of payoffs, cardinality matters heavily (Chmura et al. 2015, p. 4), along with the ordinality aspects explored here. Cardinality would gain higher importance when moving from theory and simulations to practice (potentially through behavioral experiments), a move the lack of which is a limitation to the present paper.

The fact that I did not come up with more altruistically extended payoff schemes does not mean that they are not possible, it only designates my personal limitations in creating explicable “split or steal or gift” extensions. These limitations are set and/or supported by logic and intuition, and it might be the topic of yet another paper how to prove the exhaustion of the system in terms of explicable extended altruistic payoff options.

As for the calculations of Nash equilibria, it is reasonable to embed them into a digital computational tool, for finding Nash equilibria is a task of high complexity (Sugiyama et al. 2021, 1). Sugiyama et al. name and describe three of the existing main game theory programs which can be used for Nash equilibrium computation: Gambit,

Game Theory Explorer (Antonov–von Stengel 2020), and GamePlan (Langlois n.d., Sugiyama et al. 2021, pp. 3-5), each with their own advantages and disadvantages.

The introductory pages of the above mentioned software have been looked at carefully, and Game Theory Explorer (GTE), as well as Gambit (Savani–Turocy 2023) have been chosen – due to their accessibility, clear display, and ease of use – to calculate the Nash equilibria for the four payoff schemes outlined. Through searches and functional filtering, I selected the mathematical resource page Zweig Media’s “Finite mathematics utility: game theory tool” (Waner 2007) as an additional computational tool to be applied too.

My choice of testing method, simulations, can be interpreted as a bow to “the long relationship between Game Theory and Computer Science” (Daskalakis et al. 2009, p. 89). In the interpretation of my results for practical uses, however, the limitation in the factors included in the simulation will have to be taken into account.

Looking at options (interactive game theory tool collection pages, mathematical modeling software, specialized game theory software packages) that are available,⁴ I have ultimately opted to work in the Python programming language for my simulation purposes. Other options were also excluded based on license fees, a lack (or a smaller degree) of user-friendliness, and the restrictedness in terms of the number of strategies (where only two-by-two matrices were available).

4. Results⁵

In order to distill general and comparative results from my software-assisted analyses, I constructed a summary table, with the four payoff versions as columns, and the four software tools as rows, see Table 5 below.

Table 5. Comparing results for the four payoff versions, in the four tools

	first payoff version	second payoff version	third payoff version	fourth payoff version
GTE output – the number of “extreme equilibria” (EE) and “connected component” (cc) lines	nr of EE: 6 nr of cc lines: 3	nr of EE: 4 nr of cc lines: 2	nr of EE: 5 nr of cc lines: 2	nr of EE: 7 nr of cc lines: 6
Gambit output – game tree probabilities – likeliest choice	steal	steal	steal	gift
Zweig Media output – the optimal strategy cells	gift – gift	gift – gift	gift – gift	split – gift gift – split
Python simulation results – final average payoffs	1.10875	0.8838	1.4475	0.8908

Source: own construction

⁴ And following a brief email exchange with Dr. Péter Csóka, Corvinus University of Budapest, whose PhD student has indirectly advised me to use the Python coding environment, for which advice I remain grateful.

⁵ The concrete illustration of inputs into the programs, and the more detailed results they have given will be made available upon request from the author.

Overall, the altruistic extensions have made the games more complex in terms of options and of Nash equilibria, with the fourth payoff table version yielding the highest number of “extreme equilibria” and “connected component” lines, suggesting the most challenging computation for Nash equilibria. The fourth version also stuck out in terms of the likelihood of choices, as depicted in the game trees, for it was the only version with gifting being likelier than stealing, according to the Gambit program. Moreover, the optimal strategy cells have been split, and have shifted in this version to “split – gift” and “gift – split” cells, by contrast to “gift – gift” ones in all others. The uniqueness and potential attractiveness of this table, however, has faded out in the simulations, where it has yielded low average payoffs. In regard of the simulations, the third payoff version appeared to be the most rewarding design.

Based on the results from these programs, Nash equilibria through altruistic extensions have spread more probabilistically, which has blurred the outcomes, potentially increasing payoffs in the long run (for repeated rounds in the first and third versions), but not driving choices unambiguously towards permanent “gift – gift” scenarios. This would only weakly reject my H2a (null) hypothesis, with the answer to RQ2 remaining uncertain, and up for further exploration.

5. Discussion

The boundaries between primarily and narrowly self-interested value systems and views of the interdependent and altruistically rewarding social structures can initially be difficult to overcome, from an individual perspective, and are initially “invisible”, i.e. difficult to detect in an individual from the viewpoint of others, too. This makes it a challenge to encourage the spread of altruistic behaviors, and this is why modern systems, in all of their major social spheres, can benefit from systemic tweaks that contribute to it.

In my third research question I have contemplated the potential ways in which an altruistically extended payoff scheme is more realistic than the split or steal game. The following paragraphs aim to reflect upon the corresponding hypothesis, in light of the broader analysis conducted in the paper.

As indicated earlier, real life rewards in game-like situations are non-zero-sum, not even fixed sum, rather multidimensional, with short- and long-term rewards differing, and hard to assess in the present moment. Even if one were to visualize all the dimensions and structures involved in organic and genuine decision-making, players with general human-level comprehension and modes of perception might encounter obstacles in interpreting or taking in all the information.

Seeing beyond the materialistic veils of actual rewards, beyond “split or steal”, and recognizing the additional option and rewarding strategy of “giving as a gift” in reality takes time, education, reflection, and experience – in the form of life-changing events at times. Prior to such recognitions, the lack of insight provides an explanation for the high frequency of individuals choosing to stick to the fixed (or even zero) sum areas in strategic decision making, and not to be selfless. In reality, true insights on altruism are difficult to obtain, and selfless heuristics are even harder to develop and to sustain. Once a system has shifted to more altruistic patterns, it crystallizes that “[t]here are no immediate winners or losers; the usage of mixed or randomized strategies is inadequate; and cooperation often replaces competition” (Zeleny 1975, p. 180). The process of spreading consciously

altruistic behaviors can be accelerated by encouraging them, by incorporating the rewards of altruistic choices into decisions visibly and in an articulate manner.

My intentions with the subsequent two subsections is to provide tools for shifting toward cooperative and altruistic balances. I do not claim to have covered the whole spectrum of social functioning with the selection of these major spheres, nor to offer ultimate solutions for the multidimensional networks and issues societies are interwoven with. My goal is to open the intellectual gate of enhancing social structures with smart and ethical mechanisms an inch wider than it is at present.

5.1. Implications for the corporate sphere

In this subsection, when writing about the private, corporate sector of the economy, I shift from general insights, through asymmetric and symmetric situations of individual agents, to the implications for networks and large actors.

The corporate sphere, in general, is considered to be a mostly competitive environment. The major role of competition, however, is in no direct contradiction with potential emergences of reciprocal and altruistic behaviors. The opportunities for such behaviors provide a soft, underlying ethical fabric. As Camerer writes, “reciprocity can be very important, even in competitive environments in which moral hazard is predicted” (2003, p. 96, footnote 26). Increasing the number of opportunities for altruistic behavior to be realized at one company, establishing a culture of giving through – in Camerer’s terminology – “homegrown intrinsic incentives” (2003, p. 97) in a sufficiently transparent environment is likely to fuel the spread of such acts, instead of their exploitation.

The altruistically extended payoff schemes can awaken individual players’ altruism and unite economic agents against the payoff scheme, the payoff structure of the system itself. And “[g]enerally, the more two players’ interests coincide, the more important and advantageous communication becomes” (Davis–Brams 2024). Communication contributes to the transparency of the system, thus making it safer and more welcoming for altruistic action, in a virtuous circle. The altruistic extension, however, also adds layers of complexity, and as per Davis and Brams, “[f]or games in which the players have both common and conflicting interests—in other words, in most variable-sum games, whether cooperative or noncooperative—what constitutes a solution is much harder to define and make persuasive” (Davis–Brams 2024).

Considering the financial aspects of the corporate sphere, altruistically extended payoff schemes hold analytic potential as tools for behavioral finance, for interacting with investor behavior both in active and external ways. Resulting insights could be grouped under and add to the context of financial non-linearity.

Asymmetric situations include transactions on the commodity and the labor markets. They can be contract negotiations between an individual and a business (Elridge 2023), or structures with a clearinghouse design. A concrete and specific example would be maintenance and repair contracts in the mining industry, for which strategy profiles and outcomes have already been illustrated in three-by-three payoff matrices (Pak 2007, p. 29). Also, labor market Nash equilibria may have the potential to be made more favorable through altruistic extensions. As Holt and Roth describe, “one important factor in whether such a labor market clearinghouse succeeds or fails is whether the clearinghouse is designed so that it is a Nash equilibrium for applicants and employers to

participate in a way that produces a matching of workers to jobs that is stable, in the sense that no employer and applicant who are not matched to one another would both prefer to be" (2004, p. 4001).

Asymmetry is involved in most of the transactions individuals make. Many of these transactions, as well as an increasing proportion of economic activity now takes place in the digital realm. Ever since strategic behavior has become relevant to the design of computer platforms (Daskalakis et al. 2009, p. 89), new options have opened up for charitable giving, too. During a simple online banking transaction, in an intermediary step, the customer is asked whether they wish to "support to a good cause" (OTP 2024). If the options of donating were provided in an altruistically extended payoff scheme, customer incentives to contribute may strengthen. An example in the asymmetric context would be if customers were willing to pay higher fees under the promise that their providers invest an even greater amount into charitable causes.

For symmetric situations, a classic example would be the oligopolistic race, when larger "firms selling similar products may undercut each other's price until price is driven down to cost" (Holt–Roth 2004, p. 4000). Providing a third option to firms, in the form of a market institution, augmenting the competition thereby, may divert efforts and incentives towards altruistic and worthwhile endeavors. Similar augmentations could influence the economics of auctions, and the Nash equilibria of the auction rules (Holt–Roth 2004, p. 4001).

In private economic networks, incorporating altruism in the design of market institutions has the potential of three positive effects. For one, the positive atmosphere, and the trust levels raised can induce cost effectiveness: "network trust, tied into the system of market economy selfishness, is not only capable of strengthening individuals' will to run risks, but lead[s] to an unmistakable decrease of costs in market transactions" (Hámori 2014, p. 219) Second, the altruistic institutions and behaviors may spread conveniently, independently of the sector or of the industry: "Reciprocally altruistic networks of various sizes [...] function effectively in most spheres of [the] economy" (Hámori 2014, p. 222). Third, the incorporation of altruistic incentives fosters trust, which is a personal and a market virtue at the same time: "Another significant difference between traditional exchanges and reciprocal altruism is that the lifeblood of the latter is trust, while the former relationship, actuated by self-interest, is inherently distrustful" (Hámori 2014, p. 222).

Altruistic behaviors differ between micro and macro levels, and presumably even in-between. "[A]gents of larger scale, like firms or countries, which may have to publicly deliberate before acting, may be more transparent than we are" (Kuhn 2019). They may thus have, and they may require to be incentivized to apply more sophisticated conditional strategies (Kuhn 2019).

5.2. Implications for the governmental sphere

Zooming out, to the governmental, or even to the societal level, one may draw parallels to altruistic payoff table extensions in a historical, longitudinal context too. On the one hand, extending the payoff table likens to labor division and an ever more complex economic institutional system. On the other, the increase in players' numbers would liken population

growth and integration processes (e.g. the European Union accession negotiations and procedures).

Remaining with the general and the historical intergenerational conflicts, struggles over financial assets, as well as the presently “invisible” intergenerational tension over natural resources and the environment can be inserted into the *split or steal or gift* frame. The financial aspect, regarding taxes and the public pension system, is a conflict with both sides having a present-day agency and potential impact through the political system. It is a sequential battle though, where the generation in the paying position will find itself on the receiving end, in a few decades’ time.

The phenomenon is highly dependent on demographic expectations, which may give an impression of injustice in the pensions’ context, and translate into an intergenerational moral hazard, an erosion of the pension system. Meanwhile, however, the validity of the statement that “a good society is wherein the oldest and youngest fare well” (J. Veress, personal communication, own translation, February 19, 2024) is hardly disputable, especially in a long-term, historical context.

The micro-, or individual level intertemporal economics of the longitudinal distribution of resources over one person’s lifetime, the battles between one’s past self, present self, and future self are a special type of a strategic game, where rational self-interest clashes with cognitive boundaries, expectations, uncertainties, loss aversion, and biases.

When discussing international or supranational incentive schemes, it is rather punishment than reward coming first to mind. International organizations and regionally integrated communities set legal limits upon themselves, and agree on quotas, in areas such as environment protection, migration, and military arms build-up. In terms of global spaces, international negotiation and regulation results in how nations, and groups of nations relate to, and utilize the spheres for satellites, outer space, the geographical poles, the high seas, the seabed beneath, airspace, the atmosphere, and cyberspace (Groenendijk 2024). Stemming from the anarchic state of the international community, both punishments and rewards are challenging to execute though. The *split or steal or gift* structures that groups of countries could impose on themselves would sometimes need to be inverse versions of the four payoff tables designed for this paper, because it is responsibilities and burdens being dealt with, and not financial rewards. These payoff schemes could then be enforced through smart contracts, adding a layer of validation to the agreements.

In terms of arms races, I do not see an option or space for altruism within purely bilateral relations. Isolated bilateral relations, however, can hardly be observed in the real world. Layers and dimensions of economic, geopolitical, as well as cultural interests connect states all over the globe, and it is these interdependencies to rely upon when trying to minimize national arms build-up. The virtuous trust-transparency circles hold in the global context too, making multilevel, multilateral international discourse crucial to global security.

Within a democratic nation, in present-day democratic systems, democracy itself may be described with, and modeled in *split or steal or gift* structures, albeit in a linear, rather than simultaneous manner. The split element is the agreement to the terms and the respect for rules and laws of the political system. The steal element is the fight for democratic power – there are only so many votes to be gained. In simplified terms, the

voting is zero-sum. The gift element is the act of concession – the acceptance of the other party’s win. *Ceteris paribus* – assuming the collectively self-interested and reasonable judgement of the electorate, conceding to the electoral defeat may bring benefits to the opposition even, in the form of being governed by a more capable former contestant for the next political term.

There are political and policy situations, “common social choices” (Kuhn 2019) where the rate of participation itself challenges the system, also described as volunteer dilemmas (Kuhn 2019). Three classical examples are voting (from the electorate’s perspective), vaccination, and the protection of the environment. Whenever an individual prioritizes their own personal benefits and comfort over the collective outcomes targeted, such as a high voter turnout, effective levels of vaccination, and sufficient efforts to save the natural world, their behavior aligns with the *steal* option. If they comply – vote, take on the vaccine, and make environment-friendly choices – they can be considered to have opted for *split*. The thoughts playing out in those minds could be illustrated as follows: “When we are at the threshold of adequate cooperation [...], I am better off cooperating” (Kuhn 2019). If an individual goes out of their way to get others involved as well, and to raise public awareness – those individuals are the *givers* in society.

A more peripheral example of communities encountering social choices would be risk sharing networks. “[R]isk-sharing networks based on specific forms of reciprocal altruism strengthen [...] people’s inclination to take risks under circumstances where real risks are extremely high as a consequence of underdeveloped institutional framework” (Hámori 2014, p. 219). In this scenario we see individual behavior being pushed by extreme situations, in an institutionally defective environment, and being secured by altruistic patterns of a shadow network. Whilst – at present – the positive content, the trustful side of informal relationships cannot and should not replace regulation, and professional, ethical conduct in most developed societies, it may augment contracts, transactions, and processes.

6. An outlook

The research topic I put under the magnifying glass has several open ends and points of inspiration that can be elaborated on further. From the countless potential directions and future research opportunities for the theme of altruistically extended payoff schemes I highlight four different options in this paper.

For one, game theoretical proof could be provided on the exhaustion of the system, i.e. whether the sketched payoff schemes cover all reasonable possibilities for three-by-three structures, whether they are a comprehensive set of explicitly extended altruistic payoff options (as indicated in subsection 3.1).

Second, my research methods could be seen as an analogy, or a preparation for leveling the scheme up to an n -person, multi-player game. It could prove surprising on a whole new level of complexities “to model interaction, coordination, collaboration, and collective decision-making among the agents in a multiagent system” (Rothe, 2021: 15070) with 3-by-3 payoff structures.

Third, the algorithm of extending payoff schemes with layers in an explicable manner could be generalized. This would make the game transformable not just to n -player, but n -layer versions, potentially aligning with degrees of altruism in the

dimensions and complexities of real life. If an algorithm for designing payoff structures with ever increasing players and layers were figured out, that could contribute to calculating pure and mixed Nash equilibria, potentially extending an existing process through artificially intelligent assistance. If $n > 3$ for the layers (i.e., the payoff matrix is larger than three-by-three), the illustration through real world examples becomes increasingly difficult. The models could still be applicable when paralleled with psychological dimensions, especially that the payoffs have a theoretical possibility to exist. Payoffs for subsequent n values can be designed so as to obtain the closest to the desired Nash equilibria.

Fourth, the results in this paper could be experimented on behaviorally, specifically with the three-by-three, altruistically extended payoff schemes sketched. This could either illustrate or reduce and simplify the “mental complexity” involved in games with altruistic elements, as Camerer had described (2003, p. 17). According to him, and based on the behavioral testing of simpler schemes, “[t]he experimental results suggest that it is easy to create an experimental theory world in which moral hazard is solved by norms of reciprocation” (Camerer 2003, p. 99).

7. Summary and concluding remarks

In this paper, I have embarked upon an exploration of altruistically extended payoff structures, specifically to prisoner’s dilemma style “split or steal” games. Upon adding the option of “giving as a gift” to the table, I have dived into the circumstances and consequences through three main questions, on the development and design of the payoffs, the resulting agent behavior, and the relation of these structures to reality.

I have zoomed in on and reviewed the literature in a systematic manner, starting from the basic concepts of games and Nash equilibria, moving through works that have utilized “split or steal” games as behavioral economic experiments, and closing with the relevant altruistic extensions in game theory that have been explored and analyzed already.

My methodology had two main phases. First, the development of the altruistically extended payoff structures. It was at this point that observed and described the thought process, answering my first research question on payoff matrix design, with one out of the four sub-hypotheses being rejected. Second, after introducing the available and the chosen software, for each of the four payoff tables established I have conducted a fourfold analysis (Nash equilibrium calculations through Game Theory Explorer, game tree probabilities with Gambit, optimal outcome calculations through Zweig Media’s tool, and a simulation of ten thousand rounds being played in Python). Calculation and simulation results have been summarized in the frame of a comparative analysis.

In the sections of discussion and practical implications I have shifted from theory to practice, and mostly to a larger scale. The topics were also illustrations to how incentives for cooperation can come from different conceptual directions, beyond the *sticks and carrots* duality. As regards the corporate sphere, I have moved from general insights, through asymmetric and symmetric situations of individual agents, to the implications for networks and large actors. In the governmental sector, I have discussed historical and geopolitical macro movements, intergenerational conflicts, arms buildup,

elections, vaccination issues, environmental problems, as well as risk sharing shadow-networks.

In a final main section, I have outlined four groups of possibilities for future research, potentially branching off from this paper. The noble pursuit of developing technologies and organizing principles by which individual behaviors are driven into more altruistic directions can be augmented by specific incentive scheme designs. Progress is ultimately likely to be achieved by their combination.

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