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Macroeconomic processes and expectations

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The study focuses on the problem of introducing expectations, as a subjective factor, into economic theoretical models. The basis of the model presented here is the well-known conventional Keynesian IS-LM model. It also follows that the concept of the IS-LM model can be approached as a problem or task in several ways. Also used was a discretized model encompassing three distinct categories of expectations, namely, simple, adaptive, and rational. It can be concluded that the type of expectation affects the number of stable cases in the model. The inclusion of the adaptive expectation results in the highest number of stable cases within the range of economically relevant values of the parameters studied. Numerical examples illustrate the results.

Keywords: Expectation, IS-LM model, dynamic

1. Introduction

There is no doubt that it is very important to take expectations into account in economic models. But opinions are divided on how to do it. Rational expectation[1], which was dominant for decades, has lost its luster in recent years and alternative types of expectation have come to the fore in models. Today it is much more accepted to consider alternative mechanisms for expectation formation than some years ago. In this paper, however, I return to the "old classics", even though in recent years there has also been a substantial number of papers on other types of expectation that seek to replace rational expectation.

An approach to aggregating single-value models is to assume that agents each have an individual expectation and follow a non-uniform method of constructing the expected value. That is, the environment of the agents determines not only the type of expectations, but also the method of their creation, see e.g. Curtin (2019). Due to this and similar approaches I see some problems. First of all, the problem of aggregating the expectations – as well as the aggregation of other heterogeneous individual data – is not yet solved. The second point is the context dependency of expectations. The context, the circumstances – but first and foremost their assessments – are subjective and individual elements in the process of creation of expectations. The latter depends also on a lot of other personal factors (preferences, etc.). Thus, finally there are individually colored contexts determining together with other elements related to the same individual's heterogeneous expectations. The uncertainty of uncertainties is essentially just uncertainty; therefore, the introduction of context dependency appears to be a more

[1] But it does not make anyone rational to have rational expectations. The two concepts are different. It is one thing to have a rational expectation and another to use a rational expectation type in modelling.

or less futile step – at least until the mechanisms behind the mentioned processes are more thoroughly identified. Recent research conducted by Curtin and others (for example, Fuhrer, 2017; Rots, 2017) can be considered as the beginning of new analyses.

The above ideas led to the main question for this study: what is the effect of including alternative expectation types for the same model while keeping the other assumptions based on the literature. Since expectations result from individual judgement, i.e. they can be considered subjective. I treat them as subjective factors in the models. One of the major challenges in economic modelling is how to represent the expectations of economic agents in a modelling environment. In this study, I introduce the types of expectations mostly used in the literature into the discrete, dynamic and linearized versions of the IS-LM model. In addition, I focus on aggregate expectations, i.e. I do not look at how expectations for the national economy are derived from individual expectations. Applying expectations into economic modelling raises the issue of dynamics. This means going beyond temporal statics. However, this change brings further difficulties: the dynamic IS-LM model is, even in its simplest version, quite complicated. Therefore, I examine the properties of the dynamic case first and then proceed to apply expectation types. One might ask why I work with a dynamic IS-LM model: of course, I am aware that – especially after behavioral economics' impressive results – there are a lot a complex macroeconomic models with different kinds of expectations, however, first of all, I was looking here for a model whose basic version is widely known. Second, it is a model very close to Keynes' fundamental idea that psychology is an important element of almost all economic decisions, implying that expectations have to be taken in account not only when financial decisions are made but also in the cases of deciding any other economic problem (Keynes, 1937). Moreover, we should not forget that there are many studies and theories that incorporate expectations into such a dynamic IS-LM model (Navarro–Tomé, 2022; Altar, 2008; Szomolányi et al., 2016). A dynamic macro model requires simplifications to make it easier to use. Nevertheless, at the societal level, the subjective factor – expectations – can be included in the model and its impact on the whole economy can be assessed. Our study demonstrates how the Keynesian IS-LM model can deal with expectations in its discrete, dynamic version. Then, I examine how expectations affect the stability properties of the system. The 20th century brought significant changes in economics. As a result of social and economic changes, it became necessary to take into account factors in economic models that had previously been present had either been ignored or part of a simplistic, two-dimensional caricature which did not affect the properties of the system. A milestone in the field was the inclusion of expectations and, at the same time, dynamics.

I consider the types of expectations accepted and used in the literature to show whether the choice of expectation type affects the behavior of the model. Finally, I conclude our study with a numerical example where I derive the parameter values from the literature and the available dataset. Section 2 contains the literature review. I examine the basic, linear discrete dynamic IS-LM model in Section 3. In Section 4, I expand the basic model with expectations, then in Section

5 I demonstrate a numerical example. I summarize and indentify further research goals in Section 6.

2. Literature review

"The General Theory of Employment is a useful book; but it is neither the beginning nor the end of Dynamic Economics." (Hicks, 1937:159)

We owe the creation of the IS-LM model based on Keynesian principles to Hicks (1937). The model still exists today, although it has changed. Its success is due to the ease with which its relationships can be applied into econometric models (De Vroey, 2004a; De Vroey, 2004b). Hicks' original model was static, suggesting that he did not intend to make it dynamic. His original model is the textbook version of the IS-LM model, with some minor modifications, and has been refined over the decades to apply the aggregate production function of Modigliani, who also introduced the labor market (Modigliani, 1944).

The inclusion of an increasing number of new factors and perspectives led to the expansion of the model. The remaining equilibrium of the commodity and money markets is represented by these two relationships. At the same time, several extensions were added, including the Phillips curve (Phillips, 1958), the inclusion of rational expectations (Lucas, 1976), and the AS curve (Rule et al., 1975). Despite the fine-tuning, theoretical macroeconomists have criticized the model's lack of microeconomic foundation and behavioral consistency (King, 2000). Applied macroeconomists have criticized the model for requiring the correlation between output and capacity to be zero. The central bank rejected the notion of policy irrelevance in the model. I will focus on the dynamic version of the model and review the literature. There are several versions of the dynamic IS-LM model, one of the earliest being Torre's (1977), which includes Kaldor's investment function, meaning that the level of investment depends on both the interest rate and income. However, in the classic textbook example, the investment function depends on the interest rate and the expectations of economic agents (allowing for autonomous investment demand).

As a starting point, consider the following discrete model, based on De Cesare and Sportelli (2005):

$$Y_t - Y_{t-1} = \alpha[I(Y_{t-1}, r_{t-1}) + G_{t-1} - S(Y_{t-1}^D) - T_{t-1}]$$

$$r_t - r_{t-1} = \beta[L(Y_{t-1}, r_{t-1}) - M_{t-1}] \quad (1)$$

where

Y_t	income in the t^{th} period
r_t	nominal interest rate in the t^{th} period
$I(Y_t, r_t)$	investment function (Kaldor-type)
G_t	government expenditure in the t^{th} period
T_t	tax revenue in the t^{th} period
Y_t^D	disposable income in the t^{th} period
$S(Y_t^D)$	savings function, which is related to the disposable income
M_t	nominal money supply in the t^{th} period
$L(Y_t, r_t)$	money demand function in the t^{th} period
α	coefficient of adjustment of the commodity market, expressing how strongly interest rate changes respond to money market imbalances
β	the money market adjustment coefficient, expressing how strongly income changes respond to commodity market imbalances

In some cases, other equations are added to the above, depending on the aspects of macroeconomics that modelers emphasize on the basis of the literature. Shinashi (1981) extends the above relationships to include the Phillips curve and the government budget balance, and another study (Shinashi, 1982) works with Kaldor's non-linear investment function. In the model of Rajpal et al. (2022), in addition to the two basic equations above, the investment function, the saving function, the money demand function, and government expenditure are non-linear, and the two basic equations show the time evolution of capital accumulation and restrictions on government funds. Sportelli et al. (2014) apply the lag to the public sector but introduce the fiscal balance as a third equation. Bifurcation analysis is quite common in the continuous version of the dynamic IS-LM model, as illustrated by Cai (2005). Neri and Venturi (2007) also studied bifurcation in a nonlinear fixed price model based on Shinashi's model. The classical extensions of the original IS-LM model include the assumption of an open economy, the inclusion of the consumption function, the consideration of portfolio decisions and the incorporation of the micro-foundations of investment timing into the basic model (De Vroey, 2004a; De Vroey, 2004b).

3. The basic dynamic linear IS-LM model

I consider a closed economy and use the classical linear equations used in textbook models. Suppose that the endogenous values are income and the nominal interest rate. The other values are exogenous. I_0 is the autonomous investment and a represents the interest rate sensitivity of investment. By definition, $a \leq 0$. This means that interest rate movements and changes in the value of the investment are in opposite directions. To determine the saving function in relation to disposable income, I apply the following relations: $S_t(Y_{t-1}) = S_0 + \hat{s}Y_{t-1}^{DIS} = -C_0 + \hat{s}Y_{t-1}^{DIS}$, $Y_{t-1}^{DIS} = Y_{t-1} - T_{t-1}$, where I suppose that T_{t-1} is a net tax, i.e. this is corrected for transfers and $T_t = T_{t-1} = T_{t-2} = \cdots = T_0$. In addition, $\hat{s}$ is the savings rate, which measures the share of income that is allocated to save, $0 \leq \hat{s} < 1$. a represents the interest rate sensitivity of investments.

The equation presenting the equilibrium points in the goods market is as follows, where $0 \leq \alpha$:

$$Y_t = Y_{t-1} + \alpha[I_0 + ar_{t-1} + G_{t-1} + C_0 - \hat{s}(Y_{t-1} - T_0) - T_0] \tag{2}$$

The equilibrium points of the money market are determined by the following equation, assuming a linear money demand function: $(Y_{t-1}, r_{t-1}) = mY_{t-1} + kr_{t-1}$, where k is the slope of the money demand function, which shows how much a unit change in nominal interest rate results in a change in money demand ($k \leq 0$). It means that the interest rate movements and changes in the value of the money demand are in opposite directions. m shows how much a unit change in income results in a change in money demand, i.e. the slope of the money demand function ($0 \leq m < 1$). I have the following money supply function, $\frac{M_{t-1}}{P_{t-1}}$, where M_{t-1} is the nominal money supply in the $(t-1)^{\text{th}}$ period and P_{t-1} is the price level in the $(t-1)^{\text{th}}$ period. So the equation representing the equilibrium points in the money market is as follows:

$$r_t = \beta m Y_{t-1} + (1 - \beta k) r_{t-1} - \beta \frac{M_{t-1}}{P_{t-1}} \tag{3}$$

So $\hat{s} = \frac{\partial S}{\partial Y} \geq 0, k = \frac{\partial L}{\partial r} \leq 0, m = \frac{\partial L}{\partial Y} > 0, a = \frac{\partial I}{\partial r} \leq 0$. In addition, similarly to the equation of the commodity market, $0 \leq \beta$.
The system of difference equations is then the following:

$$\begin{bmatrix} Y_t \\ r_t \end{bmatrix} = \begin{bmatrix} 1 - \alpha\hat{s} & \alpha a \\ \beta m & 1 + \beta k \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha(I_0 + G_{t-1} + C_0 + (\hat{s} - 1)T_0) \\ -\beta \frac{M_{t-1}}{P_{t-1}} \end{bmatrix} \tag{4}$$

Below I calculate the conditions for the stability of the steady state equilibrium. The steady state vector is considered to be stable if two conditions are satisfied simultaneously.
First condition is that $|I - A| \neq 0$ (Galor, 2005) is the condition of the uniqueness of the steady state equilibrium, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and A is the coefficient matrix. In this case the condition is $\hat{s}k \neq -am$. The inequality is satisfied whenever the parameters are allowed to vary within the intervals specified in the model.

The second condition is that the eigenvalues of the matrix A are strictly less than 1 in absolute value (Galor, 2005). If the absolute value of either eigenvalue is greater than 1, then the steady state is unstable. If one of the eigenvalues is equal to 1, then further analysis is required to determine stability. The eigenvalues of the coefficient matrix are (λ_1, λ_2) of $A = \begin{bmatrix} 1 - \alpha\hat{s} & \alpha a \\ \beta m & 1 + \beta k \end{bmatrix}$ matrix, i.e. $(1 - \alpha\hat{s} - \lambda)(1 + \beta k - \lambda) - \alpha a \beta m = 0$. So

$$\lambda^2 + \lambda(\alpha\hat{s} - \beta k - 2) + (1 + \beta k - \alpha\hat{s} - \alpha\hat{s}\beta k - \alpha a \beta m) = 0 \tag{5}$$

When solving the quadratic equation, I get the following eigenvalues:

$$\lambda_{1,2} = \frac{\left[(2-\alpha\hat{s}+\beta k) \pm \sqrt{[(2-\alpha\hat{s}+\beta k)^2 - 4(1-\alpha\hat{s}+\beta k-\alpha\hat{s}\beta k-\alpha a\beta m)]}\right]}{2}$$

The key factors about stability are $\alpha, \hat{s}, \beta, k, m, a$. I can conclude that the problem is very complex and that stability depends on a considerable number of parameters. The dynamic IS-LM model becomes so complicated, even with linear relationships, that it is only possible to examine stability after estimating the parameters or providing concrete values. I will return to this issue in the numerical example.

4. Expanded IS-LM model with expectations

The inclusion of expectations in the IS-LM model added dynamism, as the consideration of expectations is only possible with the consideration of time. While Friedman assumed systematic, costly and easily correctable errors in the expectations of economic agents, Sargent and Wallace's new-classical model emphasized immunity to systematic errors through the incorporation of rational expectations (De Vroey, 2004a; De Vroey, 2004b). According to Krugman (1991), the structure of the economy plays an important role in determining the importance of expectations in the economic process. Agreeing with King (1993) – that incorporating expectations into a dynamic IS-LM model makes sense from the commodity market side – I apply expectations into the investment demand through the following equation: $I_t = I_0 + ar_{t-1}$. I expand the original equation by considering the autonomous investment, which refers to the investment value that does not depend on the interest rate. Let the autonomous investment – i.e. one independent from the interest rate – be a function of expected income, that is $I_0 = f\left(Y_t^{exp}\right) = I_{00} + zY_t^{exp}$, where I_{00} represents the investment independent of expected income and $0 < z \leq 1$. z is the expected income sensitivity of autonomous investment. Y_t^{exp} is the expected income for the tth time-period. This factor influences the amount of investment in the t[th] period, $I_t(I_{00}, r_{t-1}, Y_t^{exp})$. The more favorable the sentiment, the higher the investment demand will be. Conversely, the worse the sentiment, the lower the investment demand. Thus, autonomous investment has a component that depends on expected income and a component that is independent of income, namely

$$I_t = I_{00} + ar_{t-1} + zY_t^{exp} \tag{6}$$

Below, I examine how different types of expectations influence the outcome of the system. The various types of expectations, focusing on our model with expected income, are as follows[2]:

[2] Although the equations seem clear, the literature suggests that there is no uniform interpretation of each type of expectation.

1. Rational: $Y_t^{exp} = E_{t-1}(Y_t \mid \Omega_{t-1})$, where Ω is the information set.
2. Adaptive: $Y_t^{exp} = Y_{t-1}^{exp} + \delta(Y_{t-1} - Y_{t-1}^{exp})$, where $0 < \delta < 1$.
3. Simple: $Y_t^{exp} = Y_{t-1}$

Including expectations only changes the IS equation as follows:

$$Y_t = Y_{t-1} + \alpha\left[I_{00} + ar_{t-1} + zY_t^{exp} + G_{t-1} + C_0 - \hat{s}(Y_{t-1} - T_0) - T_0\right] \quad (7)$$

Let us examine how the listed types of expectations modify the original linear dynamic IS-LM model. In all cases I examine the conditions for the existence of the steady-state vector and the stability of the steady-state equilibrium, i.e. I investigate whether it holds that $|I - A| \neq 0$ and the eigenvalues of the A matrix are between -1 and 1. In this section, I present the analytical analysis of the problems, and later, during the numerical tests, I shed light on the practical applicability.

4.1. Rational expectation

Rational expectation is an approach to economic modelling that assumes that economic agents form their expectations based on all available information. There is a large amount of literature on different interpretations of rational expectation, and the challenge for the modeler is to select the most relevant version of it. Rational expectations theory assumes that economic agents use the information available to them rationally and optimally to forecast future outcomes. As stated in King's article, "expectations about the future require that the long run and the short run are treated jointly" (King, 1993:75.) In my opinion, there is one weak point of using rational expectations in the model: the time horizon. In the case of the long run in economic terms, the concept of perfect foresight is the most appropriate one for rational expectations (in line with Dornbusch's claim that perfect foresight is the deterministic equivalent of rational expectations (Dornbusch, 1976). In this sense, perfect foresight is actually a special case where economic agents are aware of the long-term equilibrium value of the variable under consideration. However, this does not imply that they will also expect it for every period.

In this model applying rational expectations of the economic agents means that $Y_t^{exp} = Y_t$, i.e. I apply the short-run perfect foresight case of the rational expectation. This is the most common case in modelling. This version of the model is a good example of how rational expectation also has an impact, in our case on the stability of the steady state equilibrium. With this addition, the model looks as follows, where only the equation of the IS curve changes in the 2x2 case, where I must conclude that (αz) cannot be equal to 1:

$$\begin{bmatrix} Y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \frac{1-\alpha\hat{s}}{1-\alpha z} & \frac{\alpha a}{1-\alpha z} \\ \beta m & 1+\beta k \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\alpha}{1-\alpha z}(I_0 + G_{t-1} + C_0 + (\hat{s}-1)T_0) \\ -\beta \frac{M_{t-1}}{P_{t-1}} \end{bmatrix} \quad (8)$$

1st condition: $|\mathrm{I}-\mathrm{A}| = \begin{vmatrix} \frac{\alpha(z-\hat{s})}{1-\alpha z} & \frac{\alpha a}{1-\alpha z} \\ \beta m & -\beta k \end{vmatrix} = \frac{\alpha(z-\hat{s})}{1-\alpha z}(-\beta k) - \frac{\alpha a}{1-\alpha z}\beta m \rightarrow k(z-\hat{s}) \neq am$. The inequality suggests that it is a necessary condition for stability and that it is rather complicated, as household sector saving $(\hat{s})$, money market developments (k, m), the interest rate sensitivity of investment (a) and the – newly introduced – parameter (z) all play a role in the inequality.

2nd condition: For the stability, the eigenvalues of the matrix A are strictly less than 1 in absolute value. The eigenvalues are the solution of the following equation:

$$\lambda^2 + \lambda\left(\frac{1-\alpha\hat{s}}{1-\alpha z} + \beta k + 1\right) + \left(\frac{1+\beta k - \alpha\hat{s} - \alpha\hat{s}\beta k - \alpha a\beta m}{1-\alpha z}\right) = 0$$

$$\lambda_{1,2} = \frac{\left[(-1)\left(1+\beta k+\frac{1-\alpha\hat{s}}{1-\alpha z}\right) \pm \sqrt{\left[\left(1+\beta k+\frac{1-\alpha\hat{s}}{1-\alpha z}\right)^2 - 4\left(\frac{1+\beta k-\alpha\hat{s}-\alpha\hat{s}\beta k-\alpha a\beta m}{1-\alpha z}\right)\right]}\right]}{2}$$

Is summary, applying rational expectation into the discrete dynamic IS-LM model causes changing in the stability conditions of the steady-state equilibrium. It can be concluded that the inclusion of expectations in the base model is capable of changing the dynamics of the model, but in different ways. As can be seen, the conditional systems have become more complex with the inclusion of different types of expectations. For this reason, the properties of the model are further investigated by means of a numerical example.

4.2. Adaptive expectations

The adaptive expectation type is an expectation-theoretic approach to economic modelling, according to which economic agents form their expectations based on the experience of previous periods. This means that agents consider not only the current situation but also the changes observed in previous periods. In case of adaptive expectation, economic agents use past data to judge what changes will occur in the future. The model is extended to three equations, as follows.

$$\begin{bmatrix} Y_t \\ r_t \\ Y_t^{exp} \end{bmatrix} = \begin{bmatrix} 1-\alpha\hat{s}+\alpha z\delta & \alpha a & \alpha z(1-\delta) \\ \beta m & 1+\beta k & 0 \\ \delta & 0 & 1-\delta \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ r_{t-1} \\ Y_{t-1}^{exp} \end{bmatrix} + \begin{bmatrix} \alpha(I_{00}+G_{t-1}+C_0-(\hat{s}+1)T_0) \\ -\beta\frac{M_{t-1}}{P_{t-1}} \\ 0 \end{bmatrix} \tag{9}$$

Here I will also focus on the stability conditions of the steady-state equilibrium.

1st condition:

$$|I-A| = \begin{vmatrix} \alpha\hat{s}-\alpha z & \alpha a & \alpha z(1-\delta) \\ \beta m & -\beta k & 0 \\ \delta & 0 & \delta \end{vmatrix} =$$
$$(\alpha\hat{s}-\alpha z)(-\beta k)\delta + \alpha z(1-\delta)\beta k\delta - \alpha a\beta m\delta =$$
$$\alpha(\hat{s}-z)(-\beta k)\delta + \alpha z(1-\delta)\beta k\delta - \alpha a\beta m\delta.$$

So the condition is $(\hat{s}-z)(-k)+z(1-\delta)k-am \neq 0$. The interpretation of the condition is complicated because all parameters in the model affect its value.
2nd condition: The eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ are the solution of the following equation:

$$(1-\alpha\hat{s}+\alpha z\delta-\lambda)(1+\beta k-\lambda)(1+\delta-\lambda)$$
$$-\alpha z(1-\delta)(1+\beta k-\lambda)\delta - (\alpha a\beta m(1-\delta)-\lambda) = 0$$

The above relation can also only be further analyzed if I apply a numerical example to it. The equation is of degree three, so I cannot find its roots parametrically using analytical methods.

4.3. Simple expectation

The next special case is the simple expectation, i.e. $Y_t^{exp} = Y_{t-1}$. In this type of expectation, economic agents determine their expectations for the next period by observing the current value of the variable in question and expecting it for the next period. It concludes that $I_t = I_{00} + ar_{t-1} + zY_{t-1}$. The discrete difference equation system is, where $Y_t^{exp} = Y_{t-1}$:

$$\begin{bmatrix} Y_t \\ r_t \end{bmatrix} = \begin{bmatrix} 1-\alpha\hat{s}+\alpha z & \alpha a \\ \beta m & 1+\beta k \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha(I_{00}+G_{t-1}+C_0+(\hat{s}-1)T_0) \\ -\beta\frac{M_{t-1}}{P_{t-1}} \end{bmatrix} \tag{10}$$

1st condition:

$$|I-A| = \begin{vmatrix} \alpha\hat{s}-\alpha z & \alpha a \\ \beta m & -\beta k \end{vmatrix} =$$
$$(\alpha\hat{s}-\alpha z)(-\beta k) - \alpha a\beta m \rightarrow\ k(z-\hat{s}) \neq am.$$

The stability properties are influenced not only by the parameters of the original model, but also by the weight of the expected income of economic agents in the value of the autonomous investment (z).
2nd condition: The eigenvalues are the solution of the following equation:

$$\lambda^2 + \lambda(\alpha\hat{s}-\alpha z+\beta k-2) + (1+\beta k-\alpha\hat{s}-\alpha\hat{s}\beta k-\alpha a\beta m+\alpha z+\alpha z\beta k) = 0$$

$$\lambda_{1,2} = \frac{\left[(2-\alpha\hat{s}-\alpha z+\beta k) \pm \sqrt{[(2-\alpha\hat{s}-\alpha z+\beta k)^2 - 4(1-\alpha\hat{s}+\beta k-\alpha\hat{s}\beta k+\alpha a\beta m+\alpha z+\alpha z\beta k)]}\right]}{2}.$$

The result is a similarly complicated relationship as for the previous model versions. We need to further investigate the numerical examples in order to show the properties of the simple expectation type.

In this section I have shown analytically the effect on the stability of the steady state vector of including a subjective factor in the dynamic IS-LM model via the investment function: this factor is the expectation of income. I have seen that the conditions for the stability of the steady state equilibrium are already complicated in the case without expectations and with expectation too. Since no general relationships between the parameters has been discovered, I will now examine the systems of differential equations using a numerical example.

5. Numerical examples

In the first step I focus on the stability conditions of the model. For further examining the model, I present its operation through a numerical example. First, I examine the stability of the basic model where the parameters belong to the following intervals with 0.1 step-spacing: $\alpha = [0.1{:}\,0.1{:}\,0.9]$, $\beta = [0.1{:}\,0.1{:}\,0.9]$, $\hat{s} = [0.1{:}\,0.1{:}\,0.9]$, $z = [0.1{:}\,0.1{:}\,0.9]$, $\delta = [0.1{:}\,0.1{:}\,0.9]$, $m = [0.1{:}\,0.1{:}\,0.9]$, $a = [-0{,}9{:}\,0.1{:}\,-0.1]$, $k = [-0{,}9{:}\,0.1{:}\,-0.1]$. By using simulation, parameter combinations can be determined that ensure the stability of the system, using Matlab software. In the simulation, all possible combinations of parameters that satisfy both the first and the second condition were tested.

The aim of the simulation is to investigate, with the same parameter intervals, how much the steady-state equilibrium of the basic dynamic IS-LM model and its variants extended with different types of expectations changes. Table 1 shows, for parameter combinations that satisfy the first and second conditions, the percentage of stable cases in which the system remains stable as the parameter intervals begin to narrow to the economically relevant range.

Table 1. Stable cases in the basic and in the extended models

Model	1st and 2nd conditions satisfied (number of cases)[3]	$\hat{s} = [0.1{,}0.2]$ (as a percentage of stable cases)	$\hat{s} = [0.1{,}0.2]$ $m = [0.1, 0.2, 0.3, 0.4]$ (as a percentage of stable cases)	$\hat{s} = [0.1{,}0.2]$ $m = [0.1, 0.2, 0.3, 0.4]$ $k = [-0.1, -0.2, -0.3, -0.4]$ (as a percentage of stable cases)	$\hat{s} = [0.1{,}0.2]$ $m = [0.1, 0.2, 0.3, 0.4]$ $k = [-0.1, -0.2, -0.3, -0.4]$ $a = [-0.1, -0.2]$ (as a percentage of stable cases)
Basic	121256	17.9	11.9	1.5	0.8
Rational expectati on	83108	48.2	22.0	5.3	1.4
Adaptive	117436	51.9	14.5	14.54	14.54
Simple	16855	0.2	0.1	0.1	0

Source: own construction
Note: When narrowing the parameter ranges, I show the number of cases as a percentage of stable cases

[3] The total number of simulations run depended on the number of parameters included in the model version.

The first column shows the number of cases found in the simulation where the parameter combinations satisfy the first and second conditions simultaneously. The second column shows the percentage of stable cases, where I have narrowed the range of values that can be taken up by the savings rate when the first and second conditions are satisfied simultaneously. The third column shows the restriction of the value of the parameter of the money demand function. In the fourth column, I also restrict the value of the other parameter of the money demand function. In the fifth column, I also restrict the interest rate sensitivity of investment to the economically relevant range. Restricting the parameters to the economically relevant range resulted in the simulation that there are significantly fewer stable cases in the numerical example when expectations are included in the model. In addition, it can be seen across all model variants that restricting the parameters to the appropriate interval reduces the number of stable cases in each variant. At the highest percentage, the incorporation of the adaptive expectation type results in stable cases even after the narrowing.

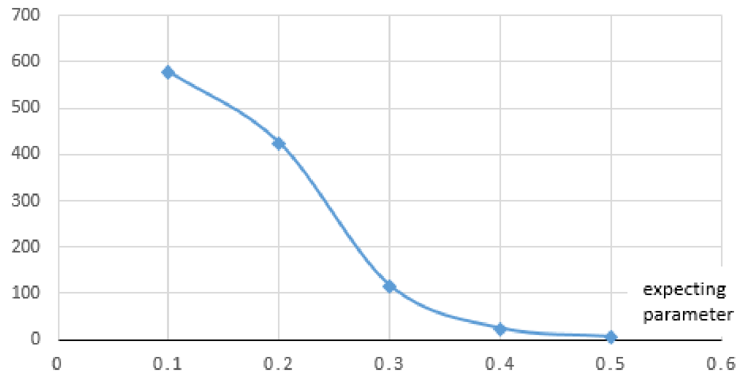
5.1. Basic model

Running a simulation with the conditions, I found 121,256 cases where both conditions are met within the specified intervals, which do not contain economic inconsistencies. I adjusted these case numbers by narrowing the interval of parameters to an economically relevant range, which is $\hat{s} = [0.1; 0.2]$, $m = [0.1: 0.1: 0.4]$, $k = [-0.1: 0.1: -0.4]$, is $a = [0.1; 0.2]$ The points were obtained in intervals that are realistic for an empirical study, for example, Oblath and Palócz (2020), Árvai and Menczel (2001), Reichel (2022). Hence, only 0.8% of the cases are economically relevant and consistent with the empirical literature. Below, I perform a simulation study of the dynamic IS-LM model augmented with certain expectation types for the same parameter intervals.

5.2. Rational expectations

During the simulations I tested all parameters which I presented above and found 83,108 cases when the first and second conditions satisfied in the same time. The economically relevant ranges mentioned in the base case are adjusted for the interest rate sensitivity of investment, money market parameters, and the savings rate. The fact that z could range between 0.1 and 0.9 during the simulation, the stable cases have values between 0.1 and 0.5. It can be shown that the higher the value of z, the fewer stable cases are found. This means that the higher share of expected income in the investment that is independent of the interest rate, the fewer stable cases can be identified.

Figure 1. In case of applying rational expectation formula in the dynamic IS-LM model



Source: own construction
Note: The expected income sensitivity parameter (z) can change the number of stable cases

Hence, the more weight given to expected income, the more unstable the equilibrium. 1.38% is the proportion of cases that were validated in the simulation based on the economics literature.

5.3. Adaptive expectations

In this case, expected income is now part of the model and can be determined based on the model's internal equations and variables. I extend the parameter intervals previously given to include the parameter for adaptive expectations, which can take values between 0 and 1. This parameter is a value that ranges from 0.1 to 0.9 with 0.1 steps and is used to represent how individuals adjust their expectations based on new information. The simulation was run by first defining the parameter combinations that satisfy the second condition within the parameter interval. Then the cases were narrowed down to those combinations that satisfy the first condition. This yielded the 117,436 stable cases. Compared to the stable cases in the base model, this number of cases is not much lower, and in fact the number of stable cases in the economically relevant parameter range is significantly higher than in the base and other extended cases. The reason for this is not only the perceived efficiency of the learning process, but it should also be noted that for this model variant the expected income is also an endogenous variable, the introduction of which has changed the properties of the baseline model.

5.4. Simple expectations

In the analytical analysis I have seen that the simple expectation case modifies the stability conditions of the steady state equilibrium compared to the baseline model. I have found significantly fewer stable cases compared to the other model variants. Moreover, in the economically relevant range, the steady state equilibrium is not stable for the given parameter intervals. The message is that this formulation almost always results in an unstable economy within economically relevant parameter ranges.

6. Conclusion

In this study, I have aimed to investigate how to apply subjective factors into an economic model; this subjective factor is the expectation. I have used the IS-LM model, its discrete, dynamic, linear version to show the effect of applying different types of expectations. The inclusion of dynamics has been justified due to the temporal evolution of expectations. First, I wrote down the basic model and examined the steady-state equilibrium conditions. Then I examined the conditions for rational expectation, adaptive expectation and simple expectation cases. To account for the large number of parameters and the complexity of their combination, simulations were carried out. I have found the most stable cases for the base model, although the number of stable cases decreased significantly as the parameters were reduced. Then, for the models augmented with each type of expectation, I have looked at how the number of stable cases evolved as the interval of parameters narrowed. The reason for the narrowing was to approximate the parameter range to the economically relevant on. For the simple expectation, I have not found stable cases within the economically relevant parameter intervals. The learning process proved to be the most efficient, i.e. the application of the adaptive expectation type. It is interesting to note that varying the expecting parameter of adaptive expectation produced roughly the same number of stable cases within the economically relevant intervals. Using adaptive expectation has resulted in a higher percentage of stability than using the most common type of expectation, rational.

The research reported on here has uncovered many new areas for investigation, which can form the basis for further studies investigating the question how the model can be tested in the case of an open economy. In addition, an empirical test of the parameters can also be carried out, possibly for a specific country. Finally, researchers can look for cases that may be relevant for economic decision-making.

References

Altar, M. (2008): A dynamic IS-LM model with adaptive expectations. *Advances in Economic and Financial Research – DOFIN Working Paper Series*, 15, Available at SSRN 1150178.

Árvai, Z. – Menczel, P. (2001): A magyar háztartások megtakarításai 1995 és 2000 között [The savings of Hungarian households between 1995 and 2000]. *Közgazdasági Szemle*, 48(2), 93-113.

Cai, J. (2005): Hopf bifurcation in the IS-LM business business cycle model with time delay. *Electronic Journal of Differential Equations*, 15, 1-6.

Curtin, R. T. (2019): *Consumer expectations: Micro foundations and macro impact*. Cambridge:Cambridge University Press.

De Cesare, L. – Sportelli, M. (2005): A dynamic IS-LM model with delayed taxation revenues. *Chaos, Solitons & Fractals*, 25(1), 233-244.

De Vroey, M. (2004a): Introduction: Seven decades of the IS-LM model. *History of Political Economy*, 36(5),·1-11, DOI: 10.1215/00182702-36-Suppl_1-1.
De Vroey, M. (2004b): *In search of lost time: The neoclassical synthesis*. Cambridge: Cambridge University Press.
Dornbusch, R. (1976): Expectations and exchange rate dynamics. *Journal of Political Economy*, 84(6), 1161-1176.
Fuhrer, J. (2017): Expectations as a source of macroeconomic persistence: Evidence from survey expectations in a dynamic macro model. *Journal of Monetary Economics*, 86, 22-35.
Galor, O. (2005): *Discrete dynamical systems*. Providence, Brown University.
Hicks, J. R. (1937): Mr. Keynes and the classics: A suggested interpretation. *Econometrica: Journal of the Econometric Society*, 5(2), 147-159.
Keynes, J. M. (1937): The general theory of employment. *The Quarterly Journal of Economics*, 51(2), 209-223.
King, R. G. (1993): Will the new keynesian macroeconomics resurrect the IS-LM model? *Journal of Economic Perspectives*, 7(1), 67-82.
King, R. G. (2000): The new IS-LM model: language, logic, and limits. *FRB Richmond Economic Quarterly*, 86(3), 45-104.
Krugman, P. (1991): History versus expectations. *The Quaterly Journal of Economics*, 106(2), 651-667.
Lucas Jr., R. E. (1976): Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1, 19-46.
Modigliani, F. (1944): Liquidity preference and the theory of interest and money. *Econometrica, Journal of the Econometric Society*, 12(1), 45-88.
Navarro, B. C. E. – Tomé, B. R. M. (2022): Structural stability analysis in a Dynamic IS-LM-AS Macroeconomic Model with inflation expectations. *International Journal of Differential Equations*, https://doi.org/10.1155/2022/5026061
Neri, U. – Venturi, B. (2007): Stability and bifurcations in IS-LM economic models. *International Review of Economics*, 54, 53-65.
Oblath, G. – Palócz, É. (2020): Gazdasági növekedés, fogyasztás és megtakarítás Magyarországon az elmúlt évtizedben. In: Kolosi, T. – Szelényi, I. – Tóth, I. Gy. (eds): *Társadalmi Riport*, Budapest, TÁRKI, 39-59.
Phillips, A. W. (1958): The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957. *Economica*, 25(100), 283-299.
Rajpal, A. – Bhatia, S. K. – Hiremath, K. R. (2022): Inspecting the stability of non-linear IS-LM model with dual time delay. *Chaos, Solitons and Fractals,* 165, 112821 https://doi.org/10.1016/j.chaos.2022.112821
Reichel, R. (2022): On the interest rate sensitivity of aggregate investment in the Euro Zone. *Asian Journal of Economic Modelling*, 10(4), 226-235.
Rots, E. (2017): Imperfect information and the house price in a general-equilibrium model. *Journal of Economic Dynamics and Control*, 83, 215-231.
Rule, O. M. S. – Sargent, T. J. – Wallace, N. (1975): "Rational" expectations, the optimal monetary instrument, and the optimal money supply rule. *Journal of Political Economy*, 83(2), 240-254.

Shinashi, G. (1981): A nonlinear dynamic model of short run fluctuations. *Review of Economic Studies,* 48, 649-656. https://doi.org/10.2307/2297204

Shinashi, G. (1982): Fluctuations in a dynamic, intermediate-run IS-LM model: Applications of the Poincaré-Bendixon theorem. *Journal of Economic Theory,* 28(2), 369-375. https://doi.org/10.1016/0022-0531(82)90068-0

Sportelli, M. – De Cesare, L. – Binetti, M. T. (2014): A dynamic IS–LM model with two time delays in the public sector. *Applied Mathematics and Computation*, 243, 728-739.

Szomolányi, K. – Lukáčik, M. – Lukáčiková, A. (2016): Dynamic IS-LM-AS Model augmented by the inflation expectations. *International Journal of Mathematical and Computational Methods*, 1, 207-213.

Torre, V. (1977): Existence of limit cycles and control in complete Keynesian system by theory of bifurcations. *Econometrica*, 45(6), 1457-1466. https://doi.org/10.2307/1912311